Basics

Intuition – What is a function?

A function relates an input to an output. It is like a machine that has an input and an output. A functions maps an input to an output

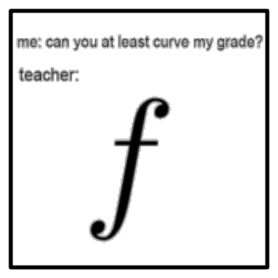
For example, let's say the function tells us to multiply by 3 and subtract 4.

Input	Function Relationship/Rule	Output/Result
0	×3 then −4	3(0) − 4 = −4
1	×3 then −4	3(1) − 4 = −1
3	×3 then −4	3(3) − 4 = 5
8	×3 then −4	3(8) − 4 = 20
x	×3 then −4	3(x) − 4 = 3x − 4

Notation

First of all, it is useful to give a function a name.

The most common name is " f ", but we can have other names like " g " ... or even " dog. "



Notation:

"$f(x) = \dots$" is the most common choice way of writing a function (we use the letter f more than any other letter)

$f(x) = 3x - 4$

function name — input — relationship/rule

In English

as: "f of x" equals $3x - 4$

This means f takes x, multiplies it by 3 and then subtracts 4

To note: We don't need to always use the letter "x" inside the bracket, it is just a place-holder, so don't get too concerned about "x", it is just there to show us where the input goes and what happens to it. It could be anything!

$f(x) = 1 - x^2 + x^3$
$f(a) = 1 - a^2 + a^3$
$f(\text{☺}) = 1 - \text{☺}^2 + \text{☺}^3$

Example 1 (very basic)

$f(x) = 3x - 4$. Find $f(5)$

Let's colour code to explain better

$f(x) = 3x - 4$

In English, $f(5)$ is saying what is the value of f when $x = 5$ which we can find by using a given rule

- Function f
- Input 5
- Relationship/rule is $3x - 4$

We **plug** the input 5 into the relationship/rule $3x - 4$ for the function f to find the output which is the solution to the question

$f(5) = 3(5) - 4$

$= 11$

Notice how we end up with an output/solution of 11

In summary, to find the value of the function at the point $x = 5$, we **plugged** the value of 5 into the relationship

A function just lates an input to an output by using its rule

Reflection: You should now understand that function relates an input to an output – it's just a rule that takes an input of x and "spits" out a y value. Just think of a function as saying, give me an x value and I'll return a y value to you.

Example 2	Example 3	Example 4:	Example 5: With harder algebra
$f(x) = x^2 + 2x - 3$ i. Find $f(3)$ ii. Solve $f(x) = 5$	$f(x) = 5 + x$ Solve $f(a) = 7$	$g(x) = \frac{1}{x+3}$ Given that $g(a) = \frac{1}{10}$, find a	$f(x) = \frac{2x}{3x+5}, g(x) = \frac{3}{x+4}$ Solve $f(x) = g(x)$
$f(x) = x^2 + 2x - 3$ i. Find $f(3)$ Replace every x in the rule for the function with 3 $f(3) = 3^2 + 2(3) - 3$ $= 12$ ii. $f(x) = 5$ Here we are given the output and need to work **backwards** Replace $f(x)$ with it's relationship/rule $x^2 + 2x - 3 = 5$ $x^2 + 2x - 8 = 0$ $(x-2)(x+4) = 0$ $x = 2, -4$	$f(x) = 5 + x$ First work out what $f(a)$ is by putting a in place of x $f(a) = 5 + a$ We can now solve $f(a) = 7$ $5 + a = 7$ $a = -2$	$g(x) = \frac{1}{x+3}$ First work out what $g(a)$ is by putting a in place of x $g(a) = \frac{1}{a+3}$ We can now solve $g(a) = \frac{1}{10}$	$f(x) = \frac{2x}{3x+5}, g(x) = \frac{3}{x+4}$ I will colour code in a less detailed way now as you should understand the topic by now $f(x) = g(x)$ $\frac{2x}{3x+5} = \frac{3}{x+4}$ Multiply both sides by $(3x+5)$ and $(x+4)$ in order to get rid of the fractions $(x+4)(2x) = 3(3x+5)$ Then we expand the brackets and use your knowledge of solving quadratics $2x^2 + 8x = 9x + 15$ $2x^2 - x - 15 = 0$ $(2x+5)(x-3) = 0$ $x = -2.5, 3?$

Composite Functions

A composite function is a function applied to another function. This typically looks like $fg(2)$ or $gf(2)$ or $fg(x)$ or $gf(x)$. The latter two might seem scary since there are no numbers, but going through example 1 part ii. below will clear this up.

Find $g(2)$ and put it into f — $fg(2)$

Put $g(x)$ and put it into f. Everywhere you see an x in f write the entire function — $fg(x)$

Always work from right to left

For composite functions we work from **right to left** (put the right inside function into the left outside function) i.e. $fg(x)$ means plug $g(x)$ into $f(x)$.

The examples below will give you the best insight on this.

Example 1:	Example 2:	Example 3:	Example 4:
$f(x) = x^3 + 1, g(x) = x - 1$ Find i. $fg(2)$ ii. $fg(x)$ iii. $(f \circ f \circ f)(1)$	$g(x) = \frac{1}{x-2}, h(x) = \frac{2x+1}{3x+4}$ Find i. $gh(1)$ ii. $hg(x)$	$f(x) = \frac{2}{x}, g(x) = \frac{x+1}{x}$ Solve $gf(a) = 3$	$f(x) = 2x - 3, g(x) = x^2 + 2$ Solve $fg(x) = gf(x)$
i. $fg(2)$ Do inside function: $g(2) = 2 - 1 = 1$ Put this into the outside function f $f(1) = 1^3 + 1 = 2$ ii. $fg(x)$ Inside function is $g(x) = x - 1$ There is nothing to simplify first, since it is terms of x. Put this into the outside function. Don't let the fact that we don't have a number now confuse you. Everywhere we see an x in the outside function we write $x - 1$ $f(x-1) = (x-1)^3 + 1$ iii. $(f \circ f \circ f)(1)$ $f(1) = 1^3 + 1 = 2$ $f(2) = 2^3 + 1 = 9$ $f(9) = 9^3 + 1 = 730$	i. $gh(1)$ Do inside function: $h(1) = \frac{3}{7}$ Put this into outside function $g\left(\frac{3}{7}\right) = \frac{1}{\frac{3}{7}-2} = \frac{1}{\frac{3-14}{7}} = -\frac{7}{11}$ ii. $hg(x)$ Nothing to do to inside function $g(x) = \frac{1}{x-2}$ $h\left(\frac{1}{x-2}\right) = \frac{2\left(\frac{1}{x-2}\right)+1}{3\left(\frac{1}{x-2}\right)+4} = \frac{2 + (x-2)}{3 + 4(x-2)} = \frac{x}{4x-5}$	$gf(a) = 3$ Let's work out LHS first $f(a) = \frac{2}{a}$ $g\left(\frac{2}{a}\right) = \frac{\frac{2}{a}+1}{\frac{2}{a}}$ Given $gf(a) = 3$ $\frac{\frac{2}{a}+1}{\frac{2}{a}} = 3$ $\Rightarrow a = 4$	Here we have a composite function on both sides Let's work out each side $fg(x) = gf(x)$ LHS: $fg(x) = 2(x^2+2) - 3 = 2x^2 + 1$ RHS: $gf(x) = (2x-3)^2 + 2 = 4x^2 - 12x + 11$ Given $fg(x) = gf(x)$ $2x^2 + 1 = 4x^2 - 12x + 11$ $2x^2 - 12x + 10 = 0 \Rightarrow x = 1, 5$

Example 5:	Example 6: Harder Algebra
$g(x) = x^2 + 3, h(x) = 2x + 2$. Solve $gh(x) = 2hg(x) + 15$	$f(x) = \frac{3x-5}{x+1}, x \in \mathbb{R}, x \neq -1$ Show that $ff(x) = \frac{x+a}{x-1}, x \in \mathbb{R}, x \neq -1, x \neq 1$. State the value of a.
$gh(x) = 2hg(x) + 15$ Let's work out LHS first: $gh(x) = (2x+2)^2 + 2$ Now let's work out the RHS: $2hg(x) + 15$ Let's first find the part $hg(x)$. We will colour code as $hg(x)$ $g(x) = x^2 + 3$ $hg(x) = 2(x^2+3) + 2$ $gh(x) = 2hg(x) + 15$ becomes $(2x+2)^2 + 2 = 2[2(x^2+3)+2] + 15$ Simplifying and solving $4x^2 + 8x + 7 = 2(2x^2 + 6 + 2) + 15$ $4x^2 + 8x + 7 = 2(2x^2 + 8) + 15$ $4x^2 + 8x + 7 = 4x^2 + 16 + 15$ $8x = 24$ $x = 3$	We calculate $ff(x)$ the same way we always do. $f(f(x)) = \frac{3(f(x)) - 5}{f(x)+1} = \frac{3\left(\frac{3x-5}{x+1}\right) - 5}{\frac{3x-5}{x+1}+1}$ **Way 1: Multiply all terms by $x+1$ to kill the fractions quickly** $= \frac{3(3x-5) - 5(x+1)}{3x - 5 + (x+1)} = \frac{9x - 15 - 5x - 5}{4x - 4} = \frac{4x - 20}{4x - 4} = \frac{x - 5}{x - 1}$ **Way 2: get a common denominator** $= \frac{\frac{3(3x-5)}{x+1} - \frac{5(x+1)}{x+1}}{\frac{3x-5}{x+1} + \frac{x+1}{x+1}} = \frac{\frac{4x-20}{x+1}}{\frac{4x-4}{x+1}} = \frac{4x-20}{4x-4} = \frac{x-5}{x-1}$ $a = -5$

An Important Deeper Understanding of Functions

Fact 1: Any function can just be replaced with the letter 'y' :

The most common name of a function is "f", but there are also other commonly used names like "g" or "h". They all mean the same thing as y. It is important to make sure you understand that we don't have to give a function a name, we can also just call it y. For example

$f(x) = x^2$ can also be written as $y = x^2$
$g(x) = x^2$ can also be written as $y = x^2$
$h(x) = x^2$ can also be written as $y = x^2$

So, going back to our very first example, $f(x) = 3x - 4$ can also be written as $y = 3x - 4$ and then graphed as usual.

Fact 2: More confusing notations used:

You may also see written in other less common ways using colons and arrows instead of brackets and equals signs. Therefore $f(x) = 3x - 4$ can be written as

$f: x \mapsto 3x - 4$ or $f: x \mapsto y$

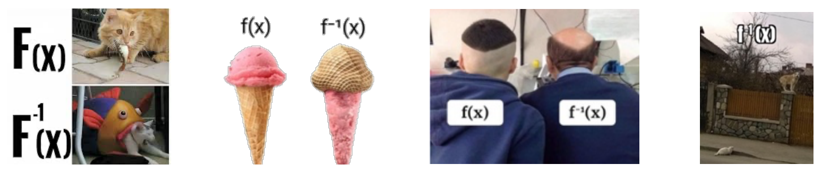
An inverse (f^{-1}) is just a reflection of the original graph (f) in the line $y = x$ hence the points of each coordinates are swapped

Inverses

Inverse Functions

Intuition

An inverse swaps things. In the case of functions it says swaps 'x' and 'y'.



Notation

The notation for the inverse is $f^{-1}(x)$. The **superscript** -1 tells us that we are finding the inverse of function f, it **does NOT mean a power of −1**

Method

- **Step 1:** Replace function with y since $f(x)$ means the same thing as y.
- **Step 2:** Swap x and y
- **Step 3:** Make y the subject
- **Step 4:** Replace the y found in step 3 with $f^{-1}(x)$

Note: Some teachers teach you to not to do step 2. Instead they go straight to step 3 and instead make x the subject in step 3 and then swap the letters at the very end. This is also ok.

So, the above steps just answer the question, given a function $f(x)$, how do I find $f^{-1}(x)$?

The difficulty with finding the inverse for most students lies in step 3. In order to make y the subject of a formula, the formula needs to be arranged and hence algebra used. Make sure you're good at the topic algebraic re-arranging/changing the subject!

Example 1	Example 2	Example 3	Example 4 Two terms with y (factorise)
$f(x) = 3x + 5$. Find $f^{-1}(x)$	$f(x) = \sqrt{2x} - 5$. Find $f^{-1}(x)$	$f(x) = 3x^2 - 5$. Find $f^{-1}(x)$	$f(x) = \frac{2-x}{4+x}$. Find $f^{-1}(x)$
Replace the function $f(x)$ with y $y = 3x + 5$ Swap x and y $y = 3x + 5$ becomes $x = 3y + 5$ Make y the subject. $x = 3y + 5$ $x - 5 = 3y$ $y = \frac{x-5}{3}$ $f^{-1}(x) = \frac{x-5}{3}$ Note: $f^{-1}(x) = \frac{-x+5}{-3}$ is also an acceptable answer	Replace the function $f(x)$ with y $y = \sqrt{2x} - 5$ Swap x and y $y = \sqrt{2x} - 5$ becomes $x = \sqrt{2y} - 5$ Make y the subject. We square both sides of the square root $x^2 = 2y - 5$ $y = \frac{x^2+5}{2}$ $f^{-1}(x) = \frac{x^2+5}{2}$ Note: $f^{-1}(x) = \frac{-x^2-5}{-2}$ is also an acceptable answer	Replace the function $f(x)$ with y $y = 3x^2 - 5$ Swap x and y $y = 3x^2 - 5$ becomes $x = 3y^2 - 5$ Make y the subject. We need to get y^2 on its own before taking the square root $3y^2 = x + 5$ $y^2 = \frac{x+5}{3} \Rightarrow y = \pm\sqrt{\frac{x+5}{3}}$ We take the positive version (unless given more info about the domain) $f^{-1}(x) = \sqrt{\frac{x+5}{3}}$	Replace function with y $y = \frac{2-x}{4+x}$ Swap x and y $x = \frac{2-y}{4+y}$ Now, make y the subject. We now have two y's. The way to deal with this is to then move all the terms with y's onto one side to be able to factorise y out and hence get y on its own. $x(4+y) = 2 - y$ $4x + xy = 2 - y$ $xy + y = 2 - 4x$ $y(x+1) = 2 - 4x$ $y = \frac{2-4x}{x+1}$ $f^{-1}(x) = \frac{2-4x}{x+1}$ Note: $\frac{4x-2}{-x-1}$ is also acceptable

Example 5: Quadratic (use quadratic formula or complete the square)	Example 6: Solving with inverse
Find the inverse of $f(x) = 1 + x - 2x^2$	$g(x) = \frac{4x}{3-x}, f(x) = 2x - 5$. Given that $x > 3$. Solve $g^{-1}(x) = f(x)$

When we swap we have to swap the x's and y's to get $x = 1 + y - 2y^2$). How then do we re-arrange for y after? We can't factorise y out from all terms like the usual harder types. We have no option but to complete the square or use quadratic formula.

Way 1: Complete the square

$x = -2\left(y^2 - \frac{1}{2}y - \frac{1}{2}\right)$

$x = -2\left[\left(y - \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{1}{2}\right]$

$= -2\left[\left(y - \frac{1}{4}\right)^2 - \frac{9}{16}\right]$

$x = -2\left(y - \frac{1}{4}\right)^2 - \frac{9}{8}$

Re-arrange for y

$2\left(y - \frac{1}{4}\right)^2 = -x + \frac{9}{8}$

$\left(y - \frac{1}{4}\right)^2 = \frac{-x + \frac{9}{8}}{2}$

$y - \frac{1}{4} = \pm\sqrt{\frac{-x + \frac{9}{8}}{2}}$

$y = \frac{1}{4} \pm \sqrt{\frac{-8x + 9}{16}} = \frac{1 \pm \sqrt{-8x+9}}{4}$

Way 2: Use Quadratic Formula (shorter)

Re-arrange first to get 0 on one side

$2y^2 - y + x - 1 = 0$

$a = 2, b = -1, c = x - 1$

$y = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(x-1)}}{2(2)}$

$y = \frac{1 \pm \sqrt{-8x+9}}{4}$

$f^{-1}(x) = \frac{1 \pm \sqrt{-8x+9}}{4}$

We take the positive version unless given more info about the domain

$\frac{1 + \sqrt{-8x+9}}{4}$

Example 6:

$g^{-1}(x) = f(x)$

First, let's work out $g^{-1}(x)$,

$y = \frac{4x}{3-x}$

$x = \frac{4y}{3-y}$

$x(3-y) = 4y$

$3x - xy = 4y$

$4y + xy = 3x$

$y(4+x) = 3x$

$y = \frac{3x}{4+x}$

Hence $g^{-1}(x) = f(x)$ becomes

$g^{-1}(x) = f(x)$

$\frac{3x}{4+x} = 2x - 5$

$3x = (2x-5)(4+x)$

$3x = 8x + 2x^2 - 20 - 5x$

$2x^2 - 20 = 0$

$x^2 = 10$

$x = \pm\sqrt{10}$

But question says $x > 3$

$x = \sqrt{10}$

Inverse and composite functions together in one question (typical harder exam questions)

Example 7:	Example 8: Harder Algebra (fraction within a fraction)	Example 9: Harder Algebra (fraction within a fraction)
$g(x) = x - 3, f(x) = x^2$. Solve $gf(x) = g^{-1}(x)$	$f(x) = \frac{4}{x-3}, g(x) = \frac{x-2}{x}$ i. Solve $fg(a) = 1$ ii. Find $f^{-1}(x)$	$f(x) = \frac{1-x}{1+x}$ Show that $ff(x) = x$
$gf(x) = g^{-1}(x)$ $gf(x) = x^2 - 3$ Now, work out the RHS, $g^{-1}(x)$, using the steps we've practised before. Set $g(x)$ to y, swap x, y, then make y the subject. $y = x - 3$ $x = y - 3$ $y = x + 3$ $g^{-1}(x) = x + 3$. So $gf(x) = g^{-1}(x)$ becomes $x^2 - 3 = x + 3$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x = 3, -2$	i. $fg(a) = 1$ $g(a) = \frac{a-2}{a}$ $f(g(a)) = \frac{4}{\frac{a-2}{a} - 3}$ $\frac{4}{\frac{a-2}{a} - 3} = 1$ There are 2 ways to proceed from here to get rid of the fractions in order to solve Way 1: Multiply all terms by a on the LHS $\frac{4}{\frac{a-2}{a} - 3} = 1$ $\frac{4a}{\left(\frac{a-2}{a}\right)a - 3a} = 1$ $\frac{4a}{a - 2 - 3a} = 1$ $\frac{4a}{-2a - 2} = 1$ $4a = -2a - 2$ $6a = -2$ $a = -\frac{1}{3}$ Way 2: Get a common denominator $\frac{4}{\frac{a-2}{a} - 3} = 1$ $\frac{4}{\frac{a-2}{a} - \frac{3a}{a}} = 1$ $\frac{4}{\frac{a-2-3a}{a}} = 1$ $\frac{4}{\frac{-2a-2}{a}} = 1$ $4 \div \frac{-2a-2}{a} = 1$ $\frac{4}{1} \times \frac{a}{-2a-2} = 1$ $\frac{4a}{-2a-2} = 1$ $4a = -2a - 2$ $a = -\frac{1}{3}$	$ff(x) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}}$ Get a common denominator $ff(x) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}}$ $= \frac{\frac{1+x}{1+x} - \frac{1-x}{1+x}}{\frac{1+x}{1+x} + \frac{1-x}{1+x}}$ Let's colour code now for ease of solving $= \frac{\frac{1+x-(1-x)}{1+x}}{\frac{1+x+(1-x)}{1+x}}$ $= \frac{\frac{2x}{1+x}}{\frac{2}{1+x}} = \frac{2x}{1+x} \div \frac{2}{1+x} = \frac{2x}{1+x} \times \frac{1+x}{2} = \frac{2x}{2} = x$ $ff(x) = x$ means that applying f twice gave the original value x, so f is its own inverse Note: If we find that $ff(x) = x$ meaning the f's cancel each other out, we say that f is **self inverse.** Applying f twice gave the original value x, so f is its own inverse. If $ff(x) = x$ we call the function f self inverse

Graphing With Function Notation

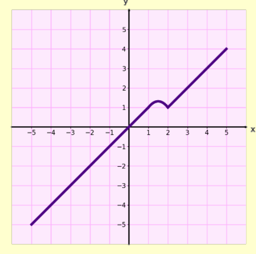
Recall $f(x)$ is the same as saying the y value of the axis on a graph. $f(x) = y$ means a function takes an input of x and returns an output y.

$f(3) = 2$ — Tells us $x = 3$, Tells us $y = 2$

$f^{-1}(3) = 2$ — Tells us $y = 3$, Tells us $x = 2$

Example 1: Basics

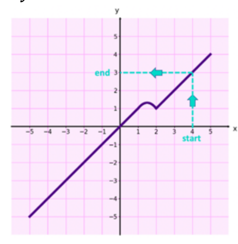
Let's look at a graphical example. Given the graph below



Use the graph to find the following

i. $f(4)$
ii. $f(-2)$
iii. Solve $f(x) = 2$

i. We look at the where the point on the graph where $x = 4$, and find the corresponding value of y



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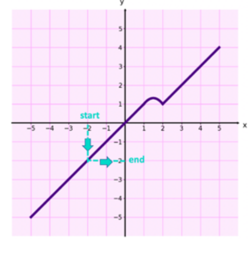
ii. We look at the where the point on the graph where $x = -2$, and find the corresponding value of y



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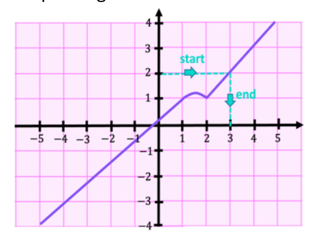
iii. We look at the where the point on the graph where y= 2, and find the corresponding value of x value



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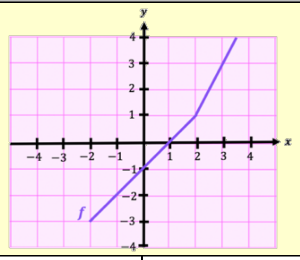
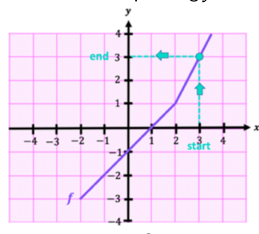
Example 2: With Composite

Consider the following graph $f(x)$
i. Write down the value of $f(3)$
ii. Write down the value of $ff(0)$



i. Recall that $f(x) = y$. This means $f(3)$ is telling us $x = 3$ and wants us to find the corresponding y

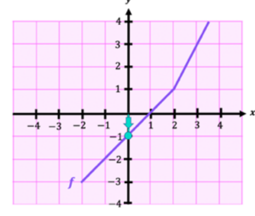


$y = 3$

So $f(3) = 3$

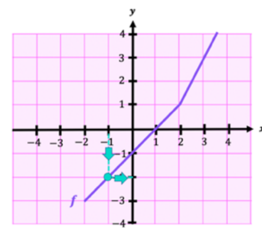
ii. $ff(0)$

Step 1: We work from right to left with composite functions



$f(0) = -1$

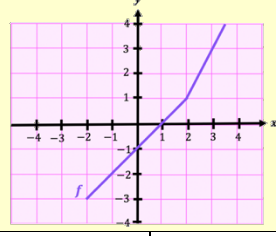
Step 2: Now we want to find $f(-1)$



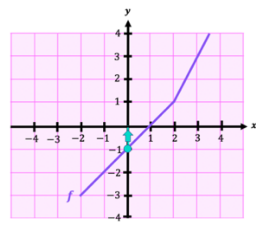
$f(-1) = -2$

Example 3 – With Inverse

Consider the following graph $f(x)$
i. Write down the value of $f^{-1}(-1)$
ii. Sketch the inverse $f^{-1}(x)$



i. Recall that $f^{-1}(y) = x$
Here is telling us $y = -1$ and wants us to find x.



Here we can see that when $y = -1$ we get $x = 0$

ii. We swap the x and y coordinates which gives us the points:
$(-2,-3), (-1,-2), (0,-1), (1,0), (2,1), (3,3)$
Become. $(-3,-2), (-2,-1), (-1,0), (0,1), (1,2), (3,3)$
Now plot these points

Harder Types Of Questions

Inverse Functions Mixed With Harder Notations

Recall, the notation for the inverse is $f^{-1}(x)$. Also recall that a function is always equal to y. This means just in the same way we can write $y = f(x)$ we can also write $y = f^{-1}(x)$.

We can also re-arrange these by taking the inverse of the other side or undoing the inverse on the other side.

$y = f(x) \Rightarrow f^{-1}(y) = x$
or
$y = f^{-1}(x) \Rightarrow f(y) = x$

For example,

$f^{-1}(2) = 5$

Tells us that

$x = 5$ and $y = 2$ in the function $f(x)$

It also tells us that

$x = 2$, and $y = 5$ in the inverse function $f^{-1}(x)$

Example 1:

Given that $f(x) = k(2 + x)$, find the value of k if $f^{-1}(-2) = -3$

Way 1: without finding the inverse (shorter method)

$f(x) = k(2 + x)$

This means the same as

$y = k(2 + x)$

Here we are given $f^{-1}(-2) = 3$

This tells us that $y = -2$ and $x = -3$ since it is the inverse and tells us the opposite to $y = f(x)$

Sub $y = -2$ and $x = -3$ into $y = k(2 + x)$:

$-2 = k(2 - 3)$
$-2 = -k$
$k = 2$

Way 2: If finding the inverse

$f(x) = k(2 + x)$

This means the same as

$y = k(2 + x)$

Find the inverse first by swapping x and y and are re-arranging for y, so we get

$x = k(2 + y)$
$x = 2k + ky$
$y = \frac{x - 2k}{k}$
$f^{-1}(x) = \frac{x - 2k}{k}$

Here we are given $f^{-1}(-2) = -3$. Sub $x = -2$ and $y = -3$ into the inverse

$-3 = \frac{-2 - 2k}{k}$
$-3k = -2 - 2k$
$k = 2$

Example 2: Harder notation

$f(x) = 2x + 1, g(x) = 5x - 3$

i. $(f \circ g^{-1})(x)$
ii. $(f^{-1} \circ g)(x)$
iii. $(f^{-1} \circ g^{-1})(x)$
iv. $(f \circ g)^{-1}(x)$

First, let's work out f^{-1} and g^{-1}, as we will need them later. We do so by setting the function to y, swapping x, y, then making y the subject.

$f^{-1}(x)$
$y = 2x + 1$
$x = 2y + 1$
$2y = x - 1$
$y = \frac{x-1}{2}$
$f^{-1}(x) = \frac{x-1}{2}$

$g^{-1}(x)$
$y = 5x - 3$
$x = 5y - 3$
$5y = x + 3$
$y = \frac{x+3}{5}$
$g^{-1}(x) = \frac{x+3}{5}$

i. $(f \circ g^{-1})(x)$

This says find $g^{-1}(x)$ and then put it into $f(x)$

This is just $fg^{-1}(x)$,

$fg^{-1}(x)$
$= f(g^{-1}(x))$
$= 2(g^{-1}(x)) + 1$
$= 2\left(\frac{x+3}{5}\right) + 1$
$= \frac{2(x+3) + 5}{5}$
$= \frac{2x + 11}{5}$

ii. $(f^{-1} \circ g)(x)$

Find $g(x)$ and then put it into $f^{-1}(x)$

This is just $f^{-1}g(x)$, we compose the two functions.

$f^{-1}g(x)$
$= f^{-1}(g(x))$
$= \frac{g(x) - 1}{2}$
$= \frac{(5x-3) - 1}{2}$
$= \frac{5x - 4}{2}$

iii. $(f^{-1} \circ g^{-1})(x)$

Find $g^{-1}(x)$ and then put it into $f^{-1}(x)$

Again, we work out $f^{-1}g^{-1}(x)$.

$f^{-1}g^{-1}(x)$
$= f^{-1}(g^{-1}(x))$
$= \frac{g^{-1}(x) - 1}{2}$
$= \frac{\frac{x+3}{5} - 1}{2}$
$= \frac{\frac{x-2}{5}}{2}$
$= \frac{x-2}{10}$

iv. $(f \circ g)^{-1}(x)$

Find $(f \circ g)(x)$ and then find the inverse of that

There are two ways of doing this. Perhaps the more straightforward way is to simply invert $(f \circ g)(x)$, which is just $fg(x)$.

$fg(x) = f(g(x))$
$= 2(g(x)) + 1$
$= 2(5x - 3) + 1$
$= 10x - 5$

Now let's invert it.

$y = 10x - 5$

Swap x and y.

$x = 10y - 5$

Make y the subject.

$10y = x + 5$
$y = \frac{x+5}{10}$

$(f \circ g)^{-1}(x) = \frac{x+5}{10}$